Abstract:

Wavelets provide a class of methods for localized signal decomposition. This is the first step in the development of localized system theories to parallel LTI (linear time invariant) systems.

Given a mother wavelet, it is needed to develop a class of operators that have the corresponding wavelets as fixed points and are linear over a given resolution. There should be criteria that one may apply for the design of optimal operators such that when applied to a signal, the result is the "closest" signal at the resolution for which the operator was designed. Then filtering or control is feasible.

This paper discusses progress in this direction for discrete signals. In particular a class of nonlinear operators (the WMMR, weighted majority with minimum range operators) that have as fixed points the PICO(N) signals (signals that are piecewise constant with the smallest length of a constant segment being N). In addition, for a fixed window width of 2N+1, the WMMR filters are linear over the Haar basis, where the length of the Haar wavelet is 2N+2. For filtering and control these filters may be optimized for edge enhancement and with respect to the ranked residuals, a figure of merit that reflects local changes in signal and noise. These properties allow the design of optimal filters/operators for filtering of PICO(N) signals under certain conditions. Applications to 1-dimensional (biological) and 2-dimensional (X-ray) data will be discussed.

Introduction:

This paper addresses concerns in dsp (discrete signal processing) and control of 1-D signals with sliding overlapping windows and adaptive transversal filters. The paper discusses problems induced by localized signals and noise that change parameter instantaneously, in particular it discusses recent results in this direction for the Haar basis.

In classical dsp, the signals and noise are assumed to be accurately represented as frequencies that are nonlocalized, i.e., of infinite extent. The associated "figures of merit" used in dsp, control, and adaptive transversal filtering and control, are $L^p$ norms that are also assumed to apply equally well to the entire signal. Therefore the classical approach in each case does not take into account instantaneous parameter changes, which are smeared out, causing loss of information about transition regions.
Wavelets have been introduced for decomposition of signals in a manner that preserves local properties. If a signal is assumed to be composed of an uncorrupted signal plus noise, where each may be represented using the same wavelet basis, then decomposition on the wavelet basis will provide a first step in filtering and control, assuming the signal and noise are composed of different frequency components. Filters/operators and figures of merit that reflect local properties when the signal and noise are accurately represented by the Haar basis are discussed below.

**Filter/Operator Structure**

The structures of the FIR (Finite Impulse Response), OS (Order Statistic), and WMMR filters are similar, the last being the object of this paper. The OS filters order the windowed values before weighting, and the WMMR filters order the windowed values in order to determine the "closest clustered" N+1 values (in a window of width 2N+1), which are then weighted. The complexity of the WMMR is therefore on the order of an OS followed by a MIN operation and the size of the class of WMMR filters is comparable to that of the FIR filters.

**Fixed Points, Linearity, and Decomposition:**

**Def.** For a given m we define $B^{2^m}$ to be the set of sequences such that each sequence is piecewise constant with the length of each constant sequence a multiple of $2^m$, and for each sequence in $B^{2^m}$ the values at times 0,1,...,$2^m-1$ are constant.

These sequences will be referred to as the PICO$'(2^m)$ sequences and note they are a subset of PICO$'(2^m)$, the class of all piecewise constant signal of length $2^m$ restricted in phase and pulse width. The following theorem is proved in [1].

**Theorem 1:** The class of WMMR filters with normalized coefficients is linear over the class of PICO$'(2^m)$ signals if $2^M \geq N+1$.

If the class of sequences is assumed to finite in extent with constant boundarys (length N+1), it is shown in [2,3] that minimal restrictions on the WMMR filters will yield a class of filters for which all fixed points are approximately PICO.

**Filtering:**
The object of this section is to design LP, HP, and BP filters for PICO signals (similar to those in the frequency domain). It is easily shown that a burst of noise of length $K<N$ is totally eliminated (using the linearity property) if the phase restrictions hold. If the phase restrictions do not hold, then a corrupted edge results and (see theorem 2) WMMR filters will restore the signal to the closest PICO(N+1) signal. The procedure for designing a high pass filter is just the opposite. A combination of the two methods would result in a BP filter.

**Theorem 2:** When WMMR filtering with weights that are normalized, nonnegative, and symmetric, then nonperfect edges asymptotically approach the closest perfect edge upon iterative application of the filter. In addition concave/convex edges are restored in one pass by the WMMR filter with median weights. (see [4,5] for more detail)

In many applications such as adaptive filtering a norm is used as part of the filtering/control process. There are many ways to make the $L^p$ norms measure only local properties. The approach used here is to order the distance of the location estimator from each of the data samples (these distances are called residuals in the statistical literature), and only consider minimizing over the residuals that are "small". For each $i$, $R_i=(r_i-s_i)^2$ is the square of the residual and is the square of the distance of the $i$th component of $r$ to the $i$th component of $s$. Therefore if we order these residuals, i.e. in the usual notation $R_{(i)}<R_{(i+1)}$, for every $i$, $N-k$ aberrant values may be ignored by minimizing over only the first $k$ ordered residuals.

**Definition:** Given a data sample $\{r_i\}$, $1<i<N$, $Q_k$ is defined as the average of all real numbers that minimize the $k$th ranked residual.

**Theorem 3:** If the windowed values are considered as data samples, $Q_{N+1}$ is equivalent to the output of the WMMR filter of length $2N+1$ and coefficients $(1/2,0,0,...,0,1/2)$. (see [3,6] for more detail)

**Application Of The WMMR Filter:**

In an application of a one-dimensional WMMR filter, C.M. Wang [7] used the LMS estimate in designing an algorithm for a robot wall follower. He compared the LMS estimate to several other robust estimates and found it to be preferred. In [8] D. Shelton applied the WMMR-MED to sinusoidal signals. He demonstrated the process yielded a viable alternative for spatial localization of a frequency within one half period in the presence of impulsive noise, i.i.d. noise,
and baseline shifts. The process was applied to biological signals (the visual evoked potential) to demonstrate identification of a response within one period whereas previous methods needed a minimum of 15 periods.

In two dimensions the WMMR-MED has been applied to X-rays with a high degree of success. Amy Glatt has applied the WMMR-MED to mammograms [9] for the selection of tumors based on size. In the application to dental X-rays [10], the goal was to teach ANN’s to recognize dental decay from digitized X-rays. The results show that an ANN preceded by a WMMR-MED filter (with a square window) gives a 20% improvement in the classification of caries over that of a trained dentist.

The WMMR has been applied to noisy X-ray images of printed circuit boards for enhancement of IC boundaries. The median or WMMR filter with the cross window will preserve corners, but not enhance edges perpendicular to the direction of travel. The one-dimensional WMMR-MED is recommended if the primary concern is to enhance the edges and smooth.

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REFERENCES


