Department of Mathematics
The University of Texas at San Antonio

Spring 2009 Seminar Series

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Time: 3:00 PM - 4:00 PM

Room: MS 2.02.52

Boundary angular derivatives of analytic self-maps of the unit disk

Let $\mathcal{S}$ denote the class of functions analytic and bounded by one in modulus on the open unit disk. These functions were characterized in terms of their Taylor coefficients at the origin by I. Schur as follows: the function $s(z) = \sum_{k=0}^{\infty} s_k z^k$ belongs to $\mathcal{S}$ if and only if the lower triangular toeplitz matrix

$$\mathbf{S}_n = \begin{bmatrix} s_0 & 0 & \cdots & 0 \\ s_1 & s_0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ s_n & \cdots & s_1 & s_0 \end{bmatrix}$$

is contractive for every integer $n \geq 0$. By a conformal change in variable, a similar result can be established for an arbitrary point $\zeta \in \mathbb{D}$ (rather than the origin).

In the talk, we will discuss a boundary analog of this result which is more of interpolation nature: given a sequence $\{c_k\}_{k=0}^{N}$ of complex numbers and given a point t_0 on the unit circle $\mathbb{T}$, we will present necessary and sufficient conditions for the existence of a function $s \in \mathcal{S}$ which admits the asymptotic expansion $s(z) = c_0 + c_1(z - t_0) + \ldots + c_N(z - t_0)^N + o(|z - t_0|^N)$ as z tends to t_0 nontangentially. The latter is equivalent to the existence of nontangential boundary limits $s_k(t_0) := \lim_{z \to t_0} \frac{s^{(k)}(z)}{k!}$ and equalities $s_k(t_0) = c_k$ for $k = 0, \ldots, N$. The case where $N = \infty$ will be also considered.

A reception will follow the talk and will be held in MS 2.02.52