

Answers to Chapter 2

Answers to Problem set 2.1

$$1. \frac{11}{2}, \frac{21}{4}, \frac{41}{8}, \frac{81}{16}, \frac{161}{32}, \frac{321}{64}, \dots$$

$$3. \frac{\sqrt{2}}{2}, 0, -\frac{\sqrt{2}}{2}, -1, -\frac{\sqrt{2}}{2}, 0, \dots$$

$$5. a_n = \frac{1}{3^n}$$

$$7. a_n = 4 + 5n$$

$$9. a_n = (-1)^n \frac{n+1}{(n+2)^2}$$

$$2. 1, 2, 6, 24, 120, 720, \dots$$

$$4. 0, 3, -2, \frac{9}{2}, -6, 11, \dots$$

$$6. a_n = \frac{1}{3n}$$

$$8. a_n = 4 + 3 \cdot (-1)^{n-1}$$

$$10. a_n = \left(-\frac{3}{5}\right)^n$$

Answers to Problem set 2.2

$$1. 0$$

$$3. \infty$$

$$5. -1$$

$$7. 8/27$$

$$9. 0$$

$$11. 0$$

$$13. 1$$

$$2. 3$$

$$4. \infty$$

$$6. \infty$$

$$8. \sqrt[3]{\frac{1}{3}}$$

$$10. 1$$

$$12. 1/2$$

$$14. 0$$

| | |
|------|------|
| 15.0 | 16.1 |
| 17.0 | 18.0 |
| 19.0 | 20.0 |
| 21.0 | 22.0 |

Answers to Problem sets 2.3 and 2.4

1. *Decreasing, Bounded*
2. *Decreasing, Bounded*
3. *Increasing, Bounded*
4. *Not monotonic, Bounded*
5. *Increasing, Unbounded (Bounded below only)*
6. *Increasing, Unbounded (Bounded below only)*
7. *Increasing, Bounded above, $\lim_{n \rightarrow \infty} a_n = 3/2$*
8. *Increasing, Bounded above, $\lim_{n \rightarrow \infty} a_n = 1/3$*
9. *Decreasing, Bounded below, $\lim_{n \rightarrow \infty} a_n = 0$*
10. *Decreasing, Bounded below, $\lim_{n \rightarrow \infty} a_n = 0$*

Answers to Problem set 2.5

- | | |
|-------------------------|-----------------------------|
| 1(a) <i>Subsequence</i> | 1(b) <i>Not subsequence</i> |
| 1(c) <i>Subsequence</i> | 1(d) <i>Not subsequence</i> |
| 2(a) <i>Subsequence</i> | 2(b) <i>Not subsequence</i> |
| 2(c) <i>Subsequence</i> | 2(d) <i>Not subsequence</i> |
3. *The given sequence has two subsequences $a_{n_k} = \frac{1}{4}$ and $b_{n_k} = -\frac{1}{4}$*
4. *The given sequence has two subsequences $a_{n_k} = 1$ and $b_{n_k} = -1$*
5. *The given sequence has two subsequences $a_{n_k} = 1$ and $b_{n_k} = 2k$*
6. *The given sequence has two subsequences $a_{n_k} = 0$ and $b_{n_k} = \frac{2}{3}$*

Answers to Chapter 3**Answers to Problem set 3.1**

- | | |
|---------------------|---------------------|
| 1. $1/3$ | 2. 3 |
| 3. $1/2$ | 4. $5/4$ |
| 5. $\frac{2e}{e-1}$ | 6. $1/11$ |
| 7. <i>diverges</i> | 8. <i>diverges</i> |
| 9. <i>diverges</i> | 10. $10/3$ |
| 11. $7/2$ | 12. <i>diverges</i> |
| 13. <i>diverges</i> | 14. <i>diverges</i> |

$$15 \text{ (a)} \sum_{n=1}^{\infty} \frac{6}{10^n}$$

$$15 \text{ (b)} \frac{2}{3}$$

$$16 \text{ (a)} 0.4 + \sum_{n=1}^{\infty} \frac{7}{10^{n+1}}$$

$$16 \text{ (b)} \frac{43}{90}$$

$$17 \text{ (a)} 2.3 + \sum_{n=1}^{\infty} \frac{21}{10^{2n+1}}$$

$$17 \text{ (b)} \frac{2298}{990}$$

$$18 \text{ (a)} 5.123 + \sum_{n=1}^{\infty} \frac{4}{10^{n+3}}$$

$$18 \text{ (b)} \frac{46111}{9000}$$

Answers to Problem set 3.2

1. *converges*

2. *diverges*

3. *converges*

4. *converges*

5. *diverges*

6. *converges*

7. *diverges*

8. *converges*

9. *diverges*

10. *converges*

11. *converges*

12. *diverges*

13. *converges*

14. *diverges*

15. *diverges*

16. *converges*

17. *diverges*

18. *diverges*

19. *converges*

20. *diverges*

21. *diverges*

22. *converges*

23. *converges*

24. *diverges*

25. *converges*

26. *diverges*

27 (a) $s_5 \approx 0.365903$. *The error is at most 0.05*

27 (b) $n > 50$

27 (c) $s \approx 0.415903$, *with error 0.00416665*

28 (a) $s_5 \approx 0.3952204$. *The error is at most 0.0067*

28 (b) $n > 16$

28 (c) $s \approx 0.4008852$, *with error 0.001*

29 (a) $s_5 \approx 0.3812364$. *The error is at most 0.01923*

29 (b) $n > 50$

29 (c) $s \approx 0.3976164$, *with error 0.00285*

30 (a) $s_5 \approx 20.1894579$. *The error is at most $6.8793.1 \cdot 10^{-7}$*

30 (b) $n \geq 4$

30 (c) $s \approx 20.189458243$, *with error $3.4 \cdot 10^{-7}$*

31. $R_{20} < 0.00125$

32. $R_{20} < 2.87 \cdot 10^{-10}$

33. $R_{20} < 4.167 \cdot 10^{-5}$

34. $R_{20} < 9.5 \cdot 10^{-7}$

Answers to Problem set 3.3

1. *converges*

2. *diverges*

3. *converges*

4. *diverges*

5. *converges*

6. *converges*

7. *converges*

8. *converges*

9. $R_5 < 0.0005787$

10. $R_5 < 3.57 \cdot 10^{-6}$

11. $R_5 < 0.00823$

12. $R_5 < 0.002304$

Answers to Problem set 3.4

1. *absolutely converges*

2. *absolutely converges*

3. *conditionally converges*

4. *absolutely converges*

5. *diverges*

6. *absolutely converges*

7. *absolutely converges*

8. *conditionally converges*

9. *absolutely converges*

10. *absolutely converges*

11. *diverges*

12. *conditionally converges*

Answers to Problem set 3.5

1. $R = 1, I = [-1, 1)$

2. $R = 2, I = [-2, 2)$

3. $R = 1/5, I = [4/5, 6/5)$

4. $R = \infty, I = (-\infty, \infty)$

5. $R = 1, I = (-3, -1)$

6. $R = 0, I = \{-2\}$

7. $R = 1, I = (1, 3)$

8. $R = \infty, I = (-\infty, \infty)$

9. $R = 0, I = \{2/3\}$

10. $R = 1/3, I = (-13/3, -11/3]$

Answers to Problem set 3.6

1. $\sum_{n=0}^{\infty} (-1)^n 2^n x^n, I = (-1/2, 1/2)$

2. $\sum_{n=0}^{\infty} 4x^{2n}, I = (-1, 1)$

3. $\sum_{n=0}^{\infty} 3(-1)^n 16^n x^{2n}, I = (-1/4, 1/4)$

4. $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{4^{n+1}}, I = (-2, 2)$

5. $\sum_{n=0}^{\infty} \left(1 + \frac{2^n}{5^{n+1}}\right) x^n, I = (-1, 1)$

6. $\sum_{n=0}^{\infty} \left[(-1)^n + \frac{1}{3^{n+1}}\right] x^{n+3}, I = (-1, 1)$

7. $-\sum_{n=0}^{\infty} \frac{(-1)^{n+1} (x-3)^n}{2^{n+1}}, I = (1, 5)$

8. $\sum_{n=0}^{\infty} \frac{(-1)^n 3^n (x-5)^n}{16^{n+1}}, I = (-1/3, 31/3)$

$$9. \sum_{n=0}^{\infty} \frac{3^n}{4^{n+1}} (x+1)^{n+1} - \sum_{n=0}^{\infty} \frac{3^n}{4^{n+1}} (x+1)^n, \quad I = (-7/3, 1/3)$$

$$10. \sum_{n=0}^{\infty} \left[\frac{1}{2^{n+1}} + (-1)^{n+1} 2^n \right] (x-1)^n, \quad I = (1/2, 3/2)$$

$$11. 1 + \sum_{n=0}^{\infty} \frac{(-1)^n}{(n+1)e^{n+1}} x^{n+1}, \quad I = (-e, e)$$

$$12. \sum_{n=0}^{\infty} \frac{x^{2n+1}}{2n+1}, \quad I = (-1, 1)$$

$$13. \sum_{n=1}^{\infty} \frac{2x^n}{n}, \quad I = (-1, 1)$$

$$14. \sum_{n=1}^{\infty} 2nx^{2n-1}, \quad I = (-1, 1)$$

$$15. \sum_{n=0}^{\infty} \frac{(-1)^{n+1} nx^{n-1}}{2^{n+1}}, \quad I = (-2, 2)$$

$$16. \sum_{n=0}^{\infty} \frac{6(-4)^n x^{2n+1}}{2n+1}, \quad I = (-1/2, 1/2)$$

Answers to Problem set 3.7

$$1. \sum_{n=0}^{\infty} \frac{3^n}{n!} x^n$$

$$2. \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+2}}{n+1}$$

$$3. \sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n+1}}{2n+1} x^{6n+3}$$

$$4. \sum_{n=0}^{\infty} \frac{(-1)^n 5^{2n+1}}{(2n+1)!} x^{2n+3}$$

5.
$$\sum_{n=0}^{\infty} \frac{(-1)^n e^2}{n!} x^{2n}$$

6.
$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{4n-3}$$

7.
$$1 + \frac{1}{x} + \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n+3}}{(2n+1)!}$$

8.
$$\sum_{n=0}^{\infty} \left[\frac{1}{n!} + \frac{(-1)^n 4^n}{(2n)!} \right] x^{2n}$$

9.
$$\sum_{n=0}^{\infty} \frac{(x-1)^n}{e^4 n!}$$

10.
$$\sum_{n=0}^{\infty} \frac{2^n (x+1)^n}{e^5 n!}$$

11.
$$\sum_{n=0}^{\infty} \frac{(-1)^{n+1} (x - \pi/2)^{2n+1}}{(2n+1)!}$$

12.
$$\sum_{n=0}^{\infty} \frac{\pi (-1)^{n+1} (x - \pi/2)^{2n}}{(2n)!}$$

Answers to Chapter 4**Answers to Problem set 4.1**

1. $M\left(\frac{3}{2}, \frac{3\sqrt{3}}{2}, 3\right)$

2. $M\left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 5\right)$

3. $M\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 8\right)$

4. $M(-2, 0, 9)$

5. $M(2, 2k\pi, -5)$

6. $M(2\sqrt{2}, \frac{\pi}{4} + 2k\pi, -7)$

7. $M(2\sqrt{3}, \frac{\pi}{3} + 2k\pi, 1)$

8. $M(2, -\frac{\pi}{3} + 2k\pi, 4)$

9. $M(0, 4, 0)$

10. $M(1, \sqrt{3}, 0)$

11. $M\left(-\frac{3\sqrt{2}}{4}, \frac{\sqrt{6}}{2}, \frac{3\sqrt{2}}{2}\right)$

12. $M\left(-\frac{1}{2}, 0, \frac{3\sqrt{2}}{2}\right)$

Answers to Problem set 4.2

1. $\vec{a} + \vec{b} = \langle 13, -6 \rangle$; $\vec{a} - \vec{b} = \langle -5, -8 \rangle$;

$5\vec{a} = \langle 20, -35 \rangle$ and $2\vec{a} - 3\vec{b} = \langle -19, -17 \rangle$

2. $\vec{a} + \vec{b} = 4\vec{i} - 3\vec{j}$; $\vec{a} - \vec{b} = 2\vec{i} - 7\vec{j}$;

$5\vec{a} = 15\vec{i} - 25\vec{j}$ and $2\vec{a} - 3\vec{b} = 3\vec{i} - 16\vec{j}$

3. $\vec{a} + \vec{b} = \langle 9, 7, -2 \rangle$; $\vec{a} - \vec{b} = \langle 1, 7, 4 \rangle$;

$5\vec{a} = \langle 25, 35, 5 \rangle$ and $2\vec{a} - 3\vec{b} = \langle -2, 14, 11 \rangle$

4. $\vec{a} + \vec{b} = 5\vec{i} - \vec{j} - 3\vec{k}$; $\vec{a} - \vec{b} = 3\vec{i} - \vec{j} + 13\vec{k}$;

$5\vec{a} = 20\vec{i} - 5\vec{j} + 25\vec{k}$ and $2\vec{a} - 3\vec{b} = 5\vec{i} - 2\vec{j} + 34\vec{k}$

5. $\vec{AB} = \langle -1, 1 \rangle$

6. $\vec{AB} = \langle 4, -1, 0 \rangle$

7. $M_1(1/2, -2, 3)$, $M_2(1, 1, 1)$, $M_3(3/2, 2, 3)$

8. $C(16, 8)$

9. $D(-3, 0, 5)$

10. $\vec{a} \cdot \vec{b} = 29$

11. $|\vec{a}| = \sqrt{46}$, $|\vec{b}| = \sqrt{21}$, $|\vec{c}| = \sqrt{59}$, $|\vec{d}| = \sqrt{18}$ and $|\vec{e}| = 1$

12. $\cos \langle \vec{a}, \vec{b} \rangle = \frac{29}{\sqrt{46}\sqrt{21}}$, $\cos \langle \vec{a}, \vec{c} \rangle = \frac{50}{\sqrt{46}\sqrt{59}}$,

$$\cos \angle (\vec{a}, \vec{e}) = \frac{3}{\sqrt{46}}$$

$$13. (2\vec{a} - 3\vec{b})(\vec{a} + 2\vec{b}) = -5, (4\vec{d} + 3\vec{e})(\vec{d} - 2\vec{e}) = 61$$

$$14. \vec{a} \times \vec{b} = 8\vec{i} - 5\vec{j} + 6\vec{k}$$

$$\vec{a} \times \vec{c} = -13\vec{i} + 6\vec{j} + 3\vec{k}$$

$$\vec{d} \times \vec{e} = 4\vec{j} - \vec{k}$$

$$\vec{e} \times \vec{d} = -4\vec{j} + \vec{k}$$

$$15. \vec{a}(\vec{b} \times \vec{c}) = -17, \vec{b}(\vec{d} \times \vec{a}) = 27$$

$$\vec{a}(\vec{b} \times \vec{d}) = 27, \vec{d}(\vec{a} \times \vec{c}) = 5$$

$$16 \text{ (a) } |AB| = 2\sqrt{6}, |BC| = \sqrt{6}, |AC| = \sqrt{26}$$

$$\text{(b) } \cos \angle (\vec{AB}, \vec{AC}) = \frac{11}{\sqrt{6}\sqrt{26}}, \cos \angle (\vec{BA}, \vec{BC}) = \frac{1}{6},$$

$$\cos \angle (\vec{CA}, \vec{CB}) = \frac{4}{\sqrt{6}\sqrt{26}}$$

$$\text{(c) } |AA_1| = \frac{\sqrt{94}}{2}, |BB_1| = \frac{\sqrt{34}}{2}, |CC_1| = \sqrt{10}$$

$$\text{(d) } A = \sqrt{35}$$

$$17. m = 4$$

$$18 \text{ (a) } V = \frac{23}{3}$$

$$18 \text{ (b) } V = 8$$

$$19 \text{ (a) } No$$

$$19 \text{ (b) } No$$

Answers to Problem set 4.3

$$1. (l) : \vec{r} = (2 + 3\lambda)\vec{i} - 2\lambda\vec{j} + (-7 + 4\lambda)\vec{k}$$

$$(l) : x = 2 + 3\lambda \quad y = -2\lambda \quad z = -7 + 4\lambda$$

$$(l) : \frac{x-2}{3} = \frac{y}{-2} = \frac{z+7}{4}$$

$$2. (l) : \vec{r} = (5 + 2\lambda)\vec{i} + 8\vec{j} - 7\lambda\vec{k}$$

$$(l) : x = 5 + 2\lambda \quad y = 8 \quad z = -7\lambda$$

$$(l) : \frac{x-5}{2} = \frac{z}{-7}, \quad y = 8$$

$$3. (l) : \vec{r} = (-3 - 4\lambda)\vec{i} + (1 + 5\lambda)\vec{j} + (5 + 7\lambda)\vec{k}$$

$$(l) : x = -3 - 4\lambda \quad y = 1 + 5\lambda \quad z = 5 + 7\lambda$$

$$(l) : \frac{x+3}{-4} = \frac{y-1}{5} = \frac{z-5}{7}$$

$$4. (l) : \vec{r} = (1 - 5\lambda)\vec{i} - 7\lambda\vec{j} + (-5 + 6\lambda)\vec{k}$$

$$(l) : x = 1 - 5\lambda \quad y = -7\lambda \quad z = -5 + 6\lambda$$

$$(l) : \frac{x-1}{-5} = \frac{y}{-7} = \frac{z+5}{6}$$

$$5. (l) : \vec{r} = \lambda(\vec{i} + 2\vec{j} + 3\vec{k})$$

$$(l) : x = \lambda \quad y = 2\lambda \quad z = 3\lambda$$

$$(l) : \frac{x}{1} = \frac{y}{2} = \frac{z}{3}$$

$$6. (l) : \vec{r} = (2 - \lambda)\vec{i} - 3\lambda\vec{j} + (-3 + 4\lambda)\vec{k}$$

$$(l) : x = 2 - \lambda \quad y = -3\lambda \quad z = -3 + 4\lambda$$

6. $(l) : \frac{x-2}{-1} = \frac{y}{-3} = \frac{z+3}{4}$

7. $M_z(-7/2, -11/4, 0), \quad M_y(2, 0, 11), \quad M_x(0, -1, 7)$

8. $M_z(14, 13, 0), \quad M_y(17/4, 0, 13/4), \quad M_x(0, -17/3, 14/3)$

9. *parallel*

10. *intersect, orthogonal*

11. *skew*

12. *skew*

13. $(\alpha) : 10x + 10y - 7z - 139 = 0$

14. $(\alpha) : 3x - 5y - 2z + 63 = 0$

15. $(\alpha) : 4x + 5y - 2z + 76 = 0$

16. $(\alpha) : 73x + 27y + 76z + 502 = 0$

17. $(\alpha) : 13x - 19y + 5z - 41 = 0$

18. $(\alpha) : 4x + y - z + 16 = 0$

19. $(\alpha) : 3x - y - 5z - 9 = 0$

20. $(\alpha) : 13x - 8y + 41z - 177 = 0$

21. $\cos < (\alpha_1, \alpha_2) = \frac{3\sqrt{3}}{14}$

22. *orthogonal planes*

23. *parallel planes*

24. *equivalent planes*

25. $d = \frac{24}{13}$

26. *circle cylinder with (c) : $(y + 1)^2 + (z - \frac{1}{2})^2 = \frac{5}{4}$ and rulings parallel to $x - axis$*

27. *sphere* $(x - 3)^2 + y^2 + (z + 1)^2 = 1$

28. *elliptic cylinder with (c) : $\frac{(z-2)^2}{36} + \frac{(y+4)^2}{18} = 1$ and rulings parallel to $z - axis$*

29. *ellipsoid with $a = 4, b = 2, c = 1$*

30. *cone with vertex $V(0, 0, -1)$*

31. *hyperboloid of one sheet*

32. *elliptic paraboloid with vertex $V(0, 0, -3)$*

Answers to Chapter 5

Answers to Problem set 5.1

1. $\langle 0, 1, -1 \rangle$

2. $\langle -3, 2, 2/3 \rangle$

3. $\langle 1/12, 4 \rangle$

4. $24 - 5\sqrt{2}, \langle -6\sqrt{2} - 20, 256, -64\sqrt{2} \rangle$

Answers to Problem set 5.2

$$1. \vec{r}'(t) = -\frac{3}{t^4} \vec{i} + 7t^6 \vec{j} - \frac{2}{(2t-1)^2} \vec{k}$$

$$2. \vec{r}'(t) = \frac{3t^2+4t-3}{(3t+2)^2} \vec{i} + (2t \sin t + t^2 \cos t) \vec{j} + 2te^{t^2} \vec{k}$$

$$3. \vec{r}'(t) = 200(2t+5)^{99} \vec{i} - \frac{3}{2} \sqrt{\frac{t-1}{t+2}} \frac{1}{(t-1)^2} \vec{j}$$

$$4. \vec{r}''(t) = (-2 \sin t - t \cos t) \vec{i} + e^{t^2}(4t^3 + 6t) \vec{j}$$

$$5. \vec{r}''(t) = (-2 \cos 2t) \vec{i} + (6t - \sin t) \vec{j} + \frac{t(3+2t^2)}{(1+t^2)\sqrt{1+t^2}} \vec{k}$$

$$6. \frac{5t\sqrt[5]{t^3}}{8} \vec{i} + \frac{40^t}{\ln 40} \vec{j} + C, \text{ where } \vec{C} = c_1 \vec{i} + c_2 \vec{j}$$

$$7. (-\cos t - \cot t) \vec{i} + \frac{1}{3} \ln(3t+7) \vec{j} + \frac{1}{2} \sin(2t-1) \vec{k} + C,$$

$$\text{where } \vec{C} = c_1 \vec{i} + c_2 \vec{j} + c_3 \vec{k}$$

$$8. (\frac{1}{3}te^{3t} - \frac{1}{9}e^{3t}) \vec{i} + (\frac{1}{4}t^4 \ln t - \frac{1}{16}t^4) \vec{j} + [t \tan^{-1}t - \frac{1}{2} \ln(1+t^2)] \vec{k} + C,$$

$$\text{where } \vec{C} = c_1 \vec{i} + c_2 \vec{j} + c_3 \vec{k}$$

$$9. \frac{1}{4} \vec{i} + \frac{3}{4} \vec{j} + \frac{\pi}{4} \vec{k}$$

$$10. \frac{\pi}{4} \vec{i} + \frac{e^\pi - 2}{5} \vec{j}$$

Answers to Problem set 5.3

$$1. \vec{T}(2) = \frac{48}{\sqrt{2369}} \vec{i} + \frac{8}{\sqrt{2369}} \vec{j} - \frac{1}{\sqrt{2369}} \vec{k}$$

$$2. \vec{T}(0) = \frac{1}{\sqrt{27}} \vec{i} + \frac{5}{\sqrt{27}} \vec{j} + \frac{1}{\sqrt{27}} \vec{k}$$

$$3. \vec{T}(\pi) = \frac{\sqrt{\pi}}{\sqrt{\pi+1}} \vec{i} + \frac{1}{\sqrt{\pi+1}} \vec{j}$$

$$4. x = 1 + 4\lambda \quad y = 1 + 2\lambda \quad z = 1 + \lambda$$

$$5. x = e + 2\lambda \quad y = e^3 + 3e^2\lambda \quad z = 4\sqrt{e} + \frac{2}{\sqrt{e}}\lambda$$

$$6. x = -\pi^2\lambda \quad y = 0 \quad z = -\pi - \lambda$$

$$7. \vec{N}(0) = \langle 0, -1, 0 \rangle, \quad \vec{B}(0) = \langle 4/5, 0, -3/5 \rangle$$

$$\text{Normal plane at } M_1(0, 3, 0) : 3x + 4z = 0$$

$$\text{Osculating plane at } M_1(0, 3, 0) : 4x - 3z = 0$$

$$\vec{N}(\pi) = \langle 0, 1, 0 \rangle, \quad \vec{B}(\pi) = \langle -4/5, 0, -3/5 \rangle$$

$$\text{Normal plane at } M_2(0, -3, 4\pi) : 3x - 4z + 16\pi = 0$$

$$\text{Osculating plane at } M_2(0, -3, 4\pi) : 4x + 3z - 12\pi = 0$$

$$\vec{N}(\pi/2) = \langle -1, 0, 0 \rangle, \quad \vec{B}(\pi) = \langle 0, -4/5, -3/5 \rangle$$

$$\text{Normal plane at } M_3(3, 0, 2\pi) : 3y - 4z + 8\pi = 0$$

$$\text{Osculating plane at } M_3(3, 0, 2\pi) : 4y + 3z - 6\pi = 0$$

$$8. \vec{N}(\pi/4) = \langle 0, 0, -1 \rangle, \quad \vec{B}(\pi/4) = \langle -\sqrt{3}/2, -1/2, 0 \rangle$$

$$\text{Normal plane at } M_1(1/2, \sqrt{3}\pi/4, 1/2) : 4x - 4\sqrt{3}y - 2 + 3\pi = 0$$

$$\text{Osculating plane at } M_1(1/2, \sqrt{3}\pi/4, 1/2) : 4\sqrt{3}x + 4y - 2\sqrt{3} - \sqrt{3}\pi = 0$$

$$\vec{N}(0) = \langle -1, 0, 0 \rangle, \quad \vec{B}(0) = \langle 0, -1/2, \sqrt{3}/2 \rangle$$

Normal plane at $M_2(1, 0, 0)$: $\sqrt{3}y + z = 0$

Osculating plane at $M_2(1, 0, 0)$: $y - \sqrt{3}z = 0$

9. 3π

10. $4\pi^2$

11. $3a/2$

12. $\frac{8}{27}(10\sqrt{10} - 1)$

13. $2\pi^2\sqrt{37}$

14. e^2

15. $\kappa_1 = \kappa(0) = a/b^2$, $\kappa_2 = \kappa(\pi) = a/b^2$

$$\kappa_3 = \kappa(\pi/2) = b/a^2, \quad \kappa_4 = \kappa(\pi/4) = \frac{2\sqrt{2}ab}{(a^2+b^2)\sqrt{a^2+b^2}}$$

16. $\kappa = \kappa(t) = 7/50$, $t \in R$

17. $\kappa_1 = \kappa(\pi) = 1/5\pi$, $\kappa_2 = \kappa(\pi/2) = 2/5\pi$

18. $\vec{V}(3) = \langle 1, -6 \rangle$, $\vec{a}(3) = \langle 0, -2 \rangle$, $|\vec{V}(3)| = \sqrt{37}$

19. $\vec{V}(1) = \langle 1, 1 \rangle$, $\vec{a}(1) = \langle -1/2, 1 \rangle$, $|\vec{V}(1)| = \sqrt{2}$

20. $\vec{V}(\pi/4) = \langle -\frac{3\sqrt{2}}{4}, \frac{3\sqrt{2}}{4}, -2 \rangle$, $\vec{a}(\pi/4) = \langle \frac{3\sqrt{2}}{4}, \frac{3\sqrt{2}}{4}, 0 \rangle$,

$$|\vec{V}(\pi/4)| = 5/2$$

21. $\vec{V}(\pi/2) = \langle 0, -1, 0 \rangle$, $\vec{a}(\pi/2) = \langle 2, 0, -1 \rangle$,

$$|\vec{V}(\pi/2)| = 1$$

Answers to Chapter 6

Answers to Problem sets 6.1 and 6.2

1. $(x, y) \in \mathbb{R}^2$

2. $(x, y) \in \mathbb{R}^2$

3. $x^2 + y^2 \neq 9$

4. $y \neq \pm 1, y^2 \leq x$

5. $x > y$

6. $x^2 + y^2 + z^2 \leq 2$

7. *Graph*8. *Graph*9. *Graph*10. *Graph*

11. -9

12. $\pi^2\sqrt{2}/8$

13. *does not exist*14. *does not exist*

15. 0

16. 0

17. e

18. *does not exist*

19. $(x, y) \in \mathbb{R}^2$

20. $x \neq y$

21. $x^2 + y^2 \neq z^2$

22. $(x, y, z) \in \mathbb{R}^3$

Answers to Problem set 6.3

1. $f_x = 3x^2 + 5y^4 - 6xy,$

$f_y = -2y + 20xy^3 - 3x^2$

2. $f_x = 1 + \frac{1}{y},$

$f_y = 1 - \frac{x}{y^2}$

3. $f_x = 4\sqrt{y} + \cos(x + 3y),$

$f_y = \frac{2x}{\sqrt{y}} + 3\cos(x + 3y)$

$$4. f_x = \frac{5x^2 - 6xy - 10y}{(5x - 3y)^2}, \quad f_y = \frac{3x^2 - 10x}{(5x - 3y)^2}$$

$$5. f_x = -\frac{2y}{(x-y)^2} e^{\frac{x+y}{x-y}}, \quad f_y = \frac{2x}{(x-y)^2} e^{\frac{x+y}{x-y}}$$

$$6. f_x = \frac{y}{3\sqrt{(xy-y^2)^2}}, \quad f_y = \frac{x-2y}{3\sqrt{(xy-y^2)^2}}$$

$$7. f_x = yz - \frac{2x}{z^2}, \quad f_y = xz - \frac{2y}{z^2}, \quad f_z = xy + \frac{2(x^2+y^2)}{z^3}$$

$$8. f_x = \frac{3x^2-z}{x^3+y^3-xz}, \quad f_y = \frac{3y^2}{x^3+y^3-xz}, \quad f_z = -\frac{x}{x^3+y^3-xz}$$

$$9. f_x = \frac{10xy+z^2}{\cos^2(5x^2y-zy^2+xz^2)}, \quad f_y = \frac{5x^2-2zy}{\cos^2(5x^2y-zy^2+xz^2)}, \quad f_z = \frac{2xz-y^2}{\cos^2(5x^2y-zy^2+xz^2)}$$

$$10. f_x = 3^{xyz} \ln 3 yz + \frac{y}{z}, \quad f_y = 3^{xyz} \ln 3 xz + \frac{x}{z}, \quad f_z = 3^{xyz} \ln 3 xy - \frac{xy}{z^2}$$

$$11. f_{xx} = 2y + \frac{2y}{x^3}, \quad f_{xy} = f_{yx} = 2x + 2y - \frac{1}{x^2}, \quad f_{yy} = 2x$$

$$12. f_{xx} = 4e^{2x+y}, \quad f_{xy} = f_{yx} = 2e^{2x+y}, \quad f_{yy} = e^{2x+y}$$

$$13. f_{xx} = -\frac{1}{x^2}, \quad f_{xy} = f_{yx} = 0, \quad f_{yy} = \frac{1}{y^2}$$

$$14. f_{xx} = -\frac{2y^3}{(x^2+y^2)^2}, \quad f_{xy} = f_{yx} = \frac{2xy^2}{(x^2+y^2)^2}, \quad f_{yy} = -\frac{2yx^2}{(x^2+y^2)^2}$$

$$15. f_{xx} = 2y, \quad f_{yy} = 2z, \quad f_{zz} = 2x$$

$$f_{xy} = f_{yx} = 2x - 3z, \quad f_{xz} = f_{zx} = 2z - 3y, \quad f_{zy} = f_{yz} = 2y - 3x$$

$$16. f_{xx} = \frac{2(y+z)}{yza^3}, \quad f_{yy} = \frac{2(x+z)}{xzy^3}, \quad f_{zz} = \frac{2(x+y)}{xyz^3}$$

$$f_{xy} = f_{yx} = \frac{1}{x^2y^2}, \quad f_{xz} = f_{zx} = \frac{1}{x^2z^2}, \quad f_{zy} = f_{yz} = \frac{1}{y^2z^2}$$

$$17. f_{xyx} = 36xy$$

$$18. f_{yyxy} = -144y \cos(3x + 2y^2) + 192y \sin(3x + 2y^2)$$

$$19. f_{xyz} = \frac{16xyz}{(x^2 + y^2 + z^2)^3}$$

Answers to Problem set 6.4

$$1. df = (3x^2 - y^2 + 2xy)dx + (x^2 - 2xy - 3y^2)dy$$

$$2. df = (se^{rs} + e^{r-s})dr + (re^{rs} - e^{r-s})ds$$

$$3. df = \frac{u}{u^2+v^2}du + \frac{v}{u^2+v^2}dv$$

$$4. df = 6x^2 \sin(x^3 + y^3)\cos(x^3 + y^3)dx + 6y^2 \sin(x^3 + y^3)\cos(x^3 + y^3)dy$$

$$5. df = (2x - y)dx + (2y - x + z)dy + (y - 2z)dz$$

$$6. df = -\frac{yz}{x^2+y^2}dx + \frac{xz}{x^2+y^2}dy + \tan^{-1}\left(\frac{y}{x}\right)dz$$

$$7. 3.96$$

$$8. 2.033125$$

$$9. 800.1 \pi$$

$$10. dz/dt = (2x + y)e^t + (x - 6y)(2t - 3)$$

$$11. dz/dt = \frac{x}{\sqrt{x^2 - 4y^2}} \cos t + \frac{4y}{\sqrt{x^2 - 4y^2}} \sin t$$

$$12. dz/dt = \frac{1}{\cos^2 \frac{x}{y}} \frac{1}{2y\sqrt{t}} - \frac{1}{\cos^2 \frac{x}{y}} \frac{x}{3y^2 \sqrt[3]{t^2}}$$

$$13. dz/dt = \frac{20t^4 \ln(2x + 3y)}{2x + 3y} + \frac{6 \ln(2x + 3y)}{(2x + 3y)\cos^2 t}$$

$$14. \partial z / \partial t = e^{2x-3y} \left(\frac{2}{\cos^2(t+s^2)} - 6t \right)$$

$$\partial z / \partial s = e^{2x-3y} \left(\frac{4s}{\cos^2(t+s^2)} + 3 \right)$$

$$15. \partial z / \partial x = 2u \ln v \cdot \left(-\frac{y}{x^2} \right) + \frac{u^2}{v} \cdot 2x$$

$$\partial z / \partial y = 2u \ln v \cdot \frac{1}{x} + \frac{u^2}{v} \cdot 2y$$

$$16. \partial z / \partial x = (2uv - v^2) \sin y - (u^2 - 2vu) y \sin x$$

$$\partial z / \partial y = (2uv - v^2) x \cos y + (u^2 - 2vu) \cos x$$

$$17. \partial u / \partial t = 2xe^s + 4yt + \frac{2z}{s}$$

$$\partial u / \partial s = 2xte^s + 4ys - \frac{2zt}{s^2}$$

$$18. y'(x) = \frac{3x^2(1 - y^2e^{x^3}) + 2xy^4}{2ye^{x^3} - 4x^2y^3}$$

$$19. y'(x) = \frac{\cos y - \cos(x+2y)}{2\cos(x+2y) + x \sin y}$$

$$20. \partial z / \partial x = \frac{3x^2 - 2yz}{2(z+xy)}, \quad \partial z / \partial y = \frac{3y^2 - 2xz}{2(z+xy)}$$

$$21. \partial z / \partial x = \frac{2\sqrt{z}\cos(x+y+z) - 2yz}{xy - 2\sqrt{z}\cos(x+y+z)}, \quad \partial z / \partial y = \frac{2\sqrt{z}\cos(x+y+z) - 2xz}{xy - 2\sqrt{z}\cos(x+y+z)}$$

Answers to Problem set 6.5

1. $\nabla f = 2xy^2 \vec{i} + (2x^2y - 3y^2) \vec{j}$

2. $\nabla f = \cos(x + 3y) \vec{i} + [3\cos(x + 3y) + 2y] \vec{j}$

3. $\nabla f = \frac{2x}{x^2 + y^2 + z^2} \vec{i} + \frac{2y}{x^2 + y^2 + z^2} \vec{j} + \frac{2z}{x^2 + y^2 + z^2} \vec{k}$

4. $\nabla f = 2xtan(y^4z) \vec{i} + \frac{4x^2y^3z}{\cos^2(y^4z)} \vec{j} + \frac{x^2y^4}{\cos^2(y^4z)} \vec{k}$

5. $D_{\vec{u}}f(9, 1) = -11/\sqrt{13}$

6. $D_{\vec{u}}f(\sqrt{\pi}, \sqrt{\pi}) = 10\sqrt{\pi}/\sqrt{53}$

7. $D_{\vec{u}}f(3, 1, 0) = 1/6\sqrt{29}$

8. $D_{\vec{u}}f(0, 1, 2) = -3/\sqrt{54}$

9. *tangent plane* : $4x + 2y - z - 5 = 0$, *normal line* : $\frac{x-2}{4} = \frac{y-1}{2} = \frac{z-5}{-1}$

10. *tangent plane* : $2x + 2y + z - 4 = 0$, *normal line* : $\frac{x-3}{4} = \frac{y}{4} = \frac{z+2}{2}$

11. *tangent plane* : $x + y - z - 1 = 0$, *normal line* : $x - 1 = y - 1 = 1 - z$

12. *tangent plane* : $x - 2y - z - 2 = 0$, *normal line* : $\frac{x-2}{2} = \frac{y-1}{-4} = \frac{z+2}{-2}$

13. $f_{min} = f(0, 0) = 0$

14. *saddle point* $(1, -1)$

15. $f_{min} = f(0, 3) = -9$

16. $f_{max} = f(1, -1) = 3$

17. $f_{min} = f(-\sqrt{2}, \sqrt{2}) = f(\sqrt{2}, -\sqrt{2}) = -8$

18. $f_{min} = f(\sqrt{6}, 6) = -864 - 12\sqrt{6}$, $f_{max} = f(-\sqrt{6}, -6) = 12\sqrt{6} + 864$,

saddle points $(-\sqrt{6}, 6)$ and $(\sqrt{6}, -6)$

19. $(1, -1, 2)$

20. $x = 2\sqrt{2}, y = 2\sqrt{2}, z = 1$

Answers to Chapter 7**Answers to Problem set 7.1**

1. $21/2$

2. $\pi/2$

3. $1/2$

4. $16/9$

5. $2/15$

6. $\sin 2/12$

7. $125/4$

8. $2/3$

9. $\pi(e - 1)$

10. $a^7\pi/7$

11. 2π

12. $-6\pi^2$

Answers to Problem set 7.2

1. $27/2$

2. $11/120$

3. $40/3$

4. $5/4$

5. $\sqrt{61}$

6. $\frac{2\pi}{3}[2\sqrt{2} - 1]$

7. 32

8. $\sqrt{2}\pi a^2$

9. $2\pi a(a - \sqrt{a^2 - b^2})$

10. $\frac{a^2}{2}(\pi - 2)$

Answers to Problem set 7.3

1. 108

2. $1/110$

3. $a^3/6$

4. 0.092

5. $416/15$

6. $e - 5/2$

7. $8a^2/9$

8. $\pi/20$

9. $\pi/5$

10. $\frac{4\pi}{3}(1 - \cos 8)$

11. 9

12. $\frac{4\pi}{3}(b^2 - a^2)^{3/2}$

13. $256\sqrt{2}/9$

14. $\frac{4\pi}{3}(b^3 - a^3)$