

Calculus III

Homework Problems

1 Problems in Chapter 2

Problem set 2.1

In Problems 1-6, list the first six elements of the sequence $\{a_n\}_{n=1}^{\infty}$.

1. $a_n = 5 + (0.5)^n$

2. $a_n = n!$

3. $\left\{ \cos \frac{n\pi}{4} \right\}_{n=1}^{\infty}$

4. $\left\{ \frac{2 + (-2)^n}{n} \right\}_{n=1}^{\infty}$

In Problems 5-10, find a formula for the general element of the sequence $\{a_n\}_{n=1}^{\infty}$.

5. $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \frac{1}{243}, \dots$

6. $\frac{1}{3}, \frac{1}{6}, \frac{1}{9}, \frac{1}{12}, \frac{1}{15}, \dots$

7. 9, 14, 19, 24, 29, ...

8. 7, 1, 7, 1, 7, ...

9. $-\frac{2}{9}, \frac{3}{16}, -\frac{4}{25}, \frac{5}{36}, \dots$

10. $-\frac{3}{5}, \frac{9}{25}, -\frac{27}{125}, \frac{81}{625}, \dots$

Problem set 2.2

In Problems 1-16 find each of the limits.

1. $\lim_{n \rightarrow \infty} \frac{7}{n+6}$

2. $\lim_{n \rightarrow \infty} \frac{9n-5}{2+3n}$

3. $\lim_{n \rightarrow \infty} n^2(n+1)$

4. $\lim_{n \rightarrow \infty} \sqrt[3]{n+3}$

5. $\lim_{n \rightarrow \infty} \frac{n^3 + 2n + 5}{6 - n - n^3}$

6. $\lim_{n \rightarrow \infty} \frac{3n^4 - 1}{3n^3 + 2}$

7. $\lim_{n \rightarrow \infty} \left(\frac{2n+1}{3n-1} \right)^3$

8. $\lim_{n \rightarrow \infty} \sqrt[3]{\frac{n^2 - 2n - 5}{3n^2 + 1}}$

9. $\lim_{n \rightarrow \infty} \frac{n!}{(n+2)!}$

10. $\lim_{n \rightarrow \infty} \tan\left(\frac{\pi n + 8}{4n}\right)$

11. $\lim_{n \rightarrow \infty} \sin\left(\frac{2n}{n^2 + 7}\right)$

12. $\lim_{n \rightarrow \infty} \cos\left(\frac{\pi n - 3}{3n}\right)$

13. $\lim_{n \rightarrow \infty} \frac{e^{1/n} - 1}{1/n}$

14. $\lim_{n \rightarrow \infty} \frac{\ln n}{n^k} \quad (k > 0)$

15. $\lim_{n \rightarrow \infty} \frac{2^n}{1 + 2^{2n}}$

16. $\lim_{n \rightarrow \infty} \frac{2^n - e^{-n}}{2^n + e^{-n}}$

In Problems 17-22, use the Squeeze Theorem and Theorem 2.3 to find each of the limits.

17. $\lim_{n \rightarrow \infty} \frac{\sin^2 n}{4^n}$

18. $\lim_{n \rightarrow \infty} \frac{\cos(3n)}{1 + \sqrt{2n}}$

19. $\lim_{n \rightarrow \infty} \frac{(-1)^n(2n+1)}{n^2 + 5}$

20. $\lim_{n \rightarrow \infty} \frac{(-1)^n n^2}{n^4 + 9}$

21. $\lim_{n \rightarrow \infty} \frac{(-1)^n \sin n}{n^3}$

22. $\lim_{n \rightarrow \infty} \frac{(-1)^n 4^n}{n!}$

Problem sets 2.3 and 2.4

In Problems 1-6, determine whether the sequence is increasing, decreasing, or not monotonic. Is the sequence bounded?

1. $a_n = \frac{1}{7^n}$

2. $a_n = \frac{1}{4n + 11}$

3. $a_n = \frac{3n-1}{5n+4}$

4. $a_n = \sin\left(n\frac{\pi}{2}\right)$

5. $a_n = 3n + \frac{1}{3n}$

6. $a_n = 2n + \ln n$

In Problems 7-10, use the Monotonic Sequence Criterion to determine whether the sequence converges or diverges. If it converges, find the limit.

7. $a_n = \frac{3n-1}{2n+5}$

8. $a_n = \frac{n-1}{3n}$

9. $a_n = \frac{3n-1}{5n^2}$

10. $a_n = \frac{(n+1)^2}{n^3}$

Problem set 2.5

In Problems 1-2, determine which of the following sequences are subsequences of a given sequences.

1. The sequence $1, 2, 3, \dots, n, \dots$ with the n th element $a_n = n$

(a) $1, 4, 9, \dots, k^2, \dots$ with the general element $b_k = k^2$

(b) $1, 2, 1, 2, \dots$, with the general element

$$b_k = \begin{cases} 1 & \text{if } k \text{ is odd} \\ 2 & \text{if } k \text{ is even} \end{cases}$$

(c) $2, 3, 5, 7, 11, 13, \dots$ when b_k is the k th prime

(d) $1, 2, 1, 4, 5, 6, 7, 8, 1, 10, \dots$, with the general element

$$b_k = \begin{cases} 1 & \text{if } k = 3^m \\ k, & \text{otherwise} \end{cases}$$

2. The sequence $1, \frac{1}{2}, \frac{1}{3}, \dots, \frac{1}{n}, \dots$ with the n th element $a_n = \frac{1}{n}$

(a) $\frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \dots, \frac{1}{k+1}, \dots$ with the general element $b_k = \frac{1}{k+1}$

(b) $\frac{1}{2}, \frac{1}{4}, \frac{1}{3}, \frac{1}{6}, \frac{1}{5}, \dots$

(c) $\frac{1}{2!}, \frac{1}{4!}, \frac{1}{6!}, \dots, \frac{1}{(2k)!}, \dots$ with the general element $b_k = \frac{1}{(2k)!}$

(d) $1, 0, \frac{1}{3}, 0, \frac{1}{5}, 0, \dots$ with the general element

$$b_k = \begin{cases} 0 & \text{if } k = 2m \\ \frac{1}{k}, & \text{otherwise} \end{cases}$$

In Problems 3-6, use the subsequences to establish the divergence of the sequences.

$$3. a_n = (-1)^n \frac{n^2}{4n^2 + 3n + 3}$$

$$4. a_n = (-1)^n \frac{n}{n+1}$$

$$5. a_n = \begin{cases} 1 & \text{if } n \text{ is odd} \\ n, & \text{if } n \text{ is even} \end{cases}$$

$$6. a_n = \frac{1 + (-1)^n}{3}$$

2 Problems in Chapter 3

Problem set 3.1

In Problems 1-14 determine whether the given series converges or diverges. If it converges, find the sum.

$$1. \sum_{n=1}^{\infty} \frac{1}{(n+2)(n+3)}$$

$$2. \sum_{n=1}^{\infty} \frac{4}{n(n+2)}$$

3.
$$\sum_{n=1}^{\infty} \frac{1}{n^2 + 3n + 2}$$

4.
$$\sum_{n=0}^{\infty} \frac{1}{5^n}$$

5.
$$\sum_{n=0}^{\infty} \frac{2}{e^n}$$

6.
$$\sum_{n=1}^{\infty} \frac{(-4)^{n-1}}{7^n}$$

7.
$$\sum_{n=0}^{\infty} 8^n$$

8.
$$\sum_{n=1}^{\infty} \frac{(-9)^{n-1}}{7^{n-1}}$$

9.
$$\sum_{n=0}^{\infty} 4 \cdot 3^{3-n} \cdot 5^n$$

10.
$$\sum_{n=0}^{\infty} \frac{4^n + 2^n}{8^n}$$

11.
$$\sum_{n=1}^{\infty} \left(\frac{1}{2n-1} - \frac{1}{2n+1} + \frac{9}{4 \cdot 10^{n-1}} \right)$$

12.
$$\sum_{n=1}^{\infty} \frac{n}{4n+7}$$

13.
$$\sum_{n=1}^{\infty} \frac{n^3 + 1}{2 - 5n^3}$$

14.
$$\sum_{n=1}^{\infty} \sqrt[n]{5}$$

In Problems 15-18 write the given decimal as:

(a) an infinite series (b) a ratio of integers

15. $0.666\dots$

16. $0.4777\dots$

17. $2.3212121\dots$

18. $5.1234444\dots$

Problem set 3.2

In Problems 1-26 test the given series for convergence.

1.
$$\sum_{n=1}^{\infty} \frac{n}{2^n}$$

2.
$$\sum_{n=1}^{\infty} \frac{9^n}{n4^n}$$

3.
$$\sum_{n=1}^{\infty} \frac{n^3 + 2n + 1}{n!}$$

5.
$$\sum_{n=1}^{\infty} \frac{n!}{n^5}$$

7.
$$\sum_{n=1}^{\infty} \left(\frac{3n^3 - 4}{2 + 5n + n^3} \right)^n$$

9.
$$\sum_{n=1}^{\infty} \left(\frac{\cos \frac{1}{n}}{\sin \frac{1}{n}} \right)^n$$

11.
$$\sum_{n=1}^{\infty} \frac{1}{n^9}$$

13.
$$\sum_{n=1}^{\infty} \frac{6 - 5\sqrt{n}}{n^4}$$

15.
$$\sum_{n=2}^{\infty} \frac{2n}{n^2 + 3}$$

17.
$$\sum_{n=1}^{\infty} \frac{11n + 3}{n(2n + 1)}$$

19.
$$\sum_{n=1}^{\infty} \frac{1}{5 + n^5}$$

21.
$$\sum_{n=1}^{\infty} \frac{5 + 7^n}{3^n}$$

23.
$$\sum_{n=1}^{\infty} \frac{n + 4}{(n + 3)^3}$$

4.
$$\sum_{n=1}^{\infty} \frac{n^n}{n!(e + 1)^n}$$

6.
$$\sum_{n=1}^{\infty} \left(\frac{2 + 5n}{9n + 1} \right)^n$$

8.
$$\sum_{n=1}^{\infty} \frac{1}{\pi^n} \left(1 + \frac{1}{n} \right)^{n^2}$$

10.
$$\sum_{n=1}^{\infty} \left(\sin^{-1} \frac{1}{n} \right)^n$$

12.
$$\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n}}$$

14.
$$\sum_{n=1}^{\infty} \frac{1}{5n + 3}$$

16.
$$\sum_{n=1}^{\infty} \frac{1}{(n - 1)^2 + 1}$$

18.
$$\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$$

20.
$$\sum_{n=1}^{\infty} \frac{1}{3n - \sqrt[3]{n}}$$

22.
$$\sum_{n=1}^{\infty} \frac{4^n}{1 + 5^n}$$

24.
$$\sum_{n=1}^{\infty} \frac{n + 7}{\sqrt[3]{n^5 + n^2}}$$

$$25. \sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^3 e^{-2n}$$

$$26. \sum_{n=1}^{\infty} \sin\left(\frac{2}{n}\right)$$

In Problems 27-30

(a) find the partial sum s_5 of the given series and estimate the error in using s_5 as an approximation to the sum of the series

(b) find a value of n so that s_n is within ε of the sum

(c) use (3.7) with $n = 5$ to give an improved estimate of the sum

$$27. \sum_{n=1}^{\infty} \frac{1}{(2n)^2}, \quad \varepsilon = 5 \cdot 10^{-3}$$

$$28. \sum_{n=1}^{\infty} \frac{1}{3n^3}, \quad \varepsilon = 7 \cdot 10^{-4}$$

$$29. \sum_{n=1}^{\infty} \frac{n}{(n^2 + 1)^2}, \quad \varepsilon = 2 \cdot 10^{-4}$$

$$30. \sum_{n=1}^{\infty} \frac{n}{2^{n^2-5}}, \quad \varepsilon = \frac{2^{-12}}{\ln 2}$$

In Problems 31-34, use the sum of the first 20 terms to approximate the sum of the series. Estimate the error.

$$31. \sum_{n=1}^{\infty} \frac{1}{n^3 + n}$$

$$32. \sum_{n=2}^{\infty} \frac{2}{1 + 3^n}$$

$$33. \sum_{n=1}^{\infty} \frac{1 + \sin^2 n}{2n^4}$$

$$34. \sum_{n=1}^{\infty} \frac{2n}{(2n + 1)2^n}$$

Problem set 3.3

In Problems 1-8, use the Alternate Series Test to determine whether the given series converges or diverges.

$$1. \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt[3]{n}}$$

$$2. \sum_{n=1}^{\infty} (-1)^{n-1} \frac{2n}{5n - 3}$$

3.
$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{n^2 + 1}$$

4.
$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{2\sqrt{n}}{1 + 3\sqrt{n}}$$

5.
$$\sum_{n=1}^{\infty} (-1)^{n-1} \sin\left(\frac{e}{n}\right)$$

6.
$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{3n}{5n^2 + 1}$$

7.
$$\sum_{n=2}^{\infty} (-1)^{n-1} \frac{1}{n\sqrt{\ln n}}$$

8.
$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{5\sqrt{n} + 1}$$

In Problems 9-12, estimate the errors in using the sum of the first 5 terms to approximate the sum of the series.

9.
$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n)^3}$$

10.
$$\sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{n^7}$$

11.
$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}n}{3^n}$$

12.
$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}n^2}{5^n}$$

Problem set 3.4

In Problems 1-14, test the given series for the absolute convergence, conditional convergence or divergence.

1.
$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{9^n}$$

2.
$$\sum_{n=1}^{\infty} \frac{(-2)^n}{n!}$$

3.
$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt[5]{n}}$$

4.
$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^7}$$

5.
$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{2n}{9 + 2n}$$

6.
$$\sum_{n=1}^{\infty} (-5)^{n-1} \frac{n}{11^n}$$

7.
$$\sum_{n=1}^{\infty} (-1)^{n-1} \left(\frac{2n^2}{n^4 - n^2 + 1} \right)^n$$

8.
$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{\sqrt{\ln n}}{n}$$

9.
$$\sum_{n=1}^{\infty} (-1)^{n-1} \left(\frac{2n-1}{6+17n} \right)^n$$

10.
$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{6n-1}{6^n}$$

11.
$$\sum_{n=1}^{\infty} (-1)^{n-1} \left(1 + \frac{1}{n} \right)^n$$

12.
$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{2n}{n^2 + 1}$$

Problem set 3.5

In Problems 1-10 find the radius of convergence and the interval of convergence of the given series.

1.
$$\sum_{n=1}^{\infty} \frac{x^n}{\sqrt[3]{n}}$$

2.
$$\sum_{n=1}^{\infty} \frac{x^n}{n2^{n-1}}$$

3.
$$\sum_{n=1}^{\infty} \frac{5^n(x-1)^n}{2n-1}$$

4.
$$\sum_{n=1}^{\infty} (-1)^n \frac{x^n}{(2n+1)!}$$

5.
$$\sum_{n=1}^{\infty} \sqrt[3]{n}(x+2)^n$$

6.
$$\sum_{n=1}^{\infty} n^n(x+2)^n$$

7.
$$\sum_{n=1}^{\infty} n^2(x-2)^n$$

8.
$$\sum_{n=1}^{\infty} \frac{(x-5)^n}{\ln^n n}$$

9.
$$\sum_{n=1}^{\infty} n!(3x-2)^n$$

10.
$$\sum_{n=1}^{\infty} \frac{(-3)^n(x+4)^n}{\sqrt{n}}$$

Problem set 3.6

In Problems 1-10, use an geometric series to find a power series representation for the function $f(x)$ centered at the given point x_0 .

1. $f(x) = \frac{1}{1+2x}, x_0 = 0$

2. $f(x) = \frac{4}{1-x^2}, x_0 = 0$

3. $f(x) = \frac{3}{1+16x^2}, x_0 = 0$

4. $f(x) = \frac{x}{4+x^2}, x_0 = 0$

5. $f(x) = \frac{3(2-x)}{2x^2-7x+5}, x_0 = 0$

6. $f(x) = \frac{4x^3}{(x+1)(3-x)}, x_0 = 0$

7. $f(x) = \frac{1}{1-x}, x_0 = 3$

8. $f(x) = \frac{1}{1+3x}, x_0 = 5$

9. $f(x) = \frac{x}{1-3x}, x_0 = -1$

10. $f(x) = \frac{4-3x}{3-7x+2x^2}, x_0 = 1$

In Problems 11-16, use the differentiation and integration of power series to find a power series representation at $x_0 = 0$ for the given function.

11. $f(x) = \ln(e+x)$

12. $f(x) = \frac{1}{2} \ln \frac{1+x}{1-x}$

13. $f(x) = \ln(1-x)^{-2}$

14. $f(x) = \frac{2x}{(1-x^2)^2}$

15. $f(x) = \frac{1}{(2+x)^2}$

16. $f(x) = 3 \tan^{-1}(2x)$

Problem set 3.7

In Problems 1-8, find a Maclaurin series representation for the given function.

1. $f(x) = e^{3x}$

2. $f(x) = \ln(1 + x^2)$

3. $f(x) = \tan^{-1}(3x^3)$

4. $f(x) = x^2 \sin(5x)$

5. $f(x) = e^{-x^2+2}$

6. $f(x) = \frac{\cos(x^2)}{x^3}$

7. $f(x) = 1 + \frac{1}{x} + \sin(x^3)$

8. $f(x) = e^{x^2} + \cos(2x)$

In Problems 9-12, find a Taylor series representation at x_0 for the given function.

9. $f(x) = e^{x-5}$, $x_0 = 1$

10. $f(x) = e^{2x-3}$, $x_0 = -1$

11. $f(x) = \cos x$, $x_0 = \frac{\pi}{2}$

12. $f(x) = \pi \sin(x - \pi)$, $x_0 = \frac{\pi}{2}$

3 Problems in Chapter 4**Problem set 4.1**

In Problems 1-4 change from cylindrical to rectangular coordinates.

1. $M(3, \frac{\pi}{3}, 3)$

2. $M(1, \frac{\pi}{6}, 5)$

3. $M(1, \frac{3\pi}{4}, 8)$

4. $M(2, \pi, 9)$

In Problems 5-8 change from rectangular to cylindrical coordinates.

5. $M(2, 0, -5)$

6. $M(2, 2, -7)$

7. $M(\sqrt{3}, 3, 1)$

8. $M(1, -\sqrt{3}, 4)$

In Problems 9-12 change from spherical to rectangular coordinates.

9. $M(4, \frac{\pi}{2}, \frac{\pi}{2})$

10. $M(2, \frac{\pi}{3}, \frac{\pi}{2})$

11. $M(3, \frac{2\pi}{3}, \frac{\pi}{4})$

12. $M(1, \pi, \frac{\pi}{6})$

Problem set 4.2

In Problems 1-4 find $\vec{a} + \vec{b}$; $\vec{a} - \vec{b}$; $5\vec{a}$ and $2\vec{a} - 3\vec{b}$ for the vectors

1. $\vec{a} = \langle 4, -7 \rangle$ and $\vec{b} = \langle 9, 1 \rangle$

2. $\vec{a} = 3\vec{i} - 5\vec{j}$ and $\vec{b} = \vec{i} + 2\vec{j}$

3. $\vec{a} = \langle 5, 7, 1 \rangle$ and $\vec{b} = \langle 4, 0, -3 \rangle$

4. $\vec{a} = 4\vec{i} - \vec{j} + 5\vec{k}$ and $\vec{b} = \vec{i} - 8\vec{k}$

In Problems 5-6 find the coordinates of the vector \overrightarrow{AB} if :

5. $A(3, -1)$ and $B(2, 0)$

6. $A(1, 7, 4)$ and $B(5, 6, 4)$

7. Given a triangle ABC with $A(1, -1, 5)$, $B(0, -3, 1)$ and $C(2, 5, 1)$. Find the coordinates of the midpoints M_1 , M_2 and M_3 of AB , BC and AC respectively.

8. Given a rectangle $ABCD$ with $A(1, 3)$, $B(13, 12)$ and $D(4, -1)$.

Find the coordinates of C .

9. Given a parallelogram $ABCD$ with $A(1, 1, 4)$, $B(2, 3, -1)$ and $C(-2, 2, 0)$. Find the coordinates of D .

Let $\vec{a} = \langle 3, 6, 1 \rangle$, $\vec{b} = \langle 1, 4, 2 \rangle$, $\vec{c} = \langle 3, 7, -1 \rangle$, $\vec{d} = \langle 1, 1, 4 \rangle$ and $\vec{e} = \langle 1, 0, 0 \rangle$. In Problems 10-15, find

10. $\vec{a} \cdot \vec{b}$

11. $|\vec{a}|$, $|\vec{b}|$, $|\vec{c}|$, $|\vec{d}|$ and $|\vec{e}|$

12. $\cos \langle \vec{a}, \vec{b} \rangle$, $\cos \langle \vec{a}, \vec{c} \rangle$, $\cos \langle \vec{a}, \vec{e} \rangle$

13. $(2\vec{a} - 3\vec{b})(\vec{a} + 2\vec{b})$, $(4\vec{d} + 3\vec{e})(\vec{d} - 2\vec{e})$

14. $\vec{a} \times \vec{b}$, $\vec{a} \times \vec{c}$, $\vec{d} \times \vec{e}$, $\vec{e} \times \vec{d}$

15. $\vec{a}(\vec{b} \times \vec{c})$, $\vec{b}(\vec{d} \times \vec{a})$, $\vec{a}(\vec{b} \times \vec{d})$, $\vec{d}(\vec{a} \times \vec{c})$

16. Given a triangle ABC with $A(3, 6, 1)$, $B(1, 2, -1)$ and $C(-1, 3, 0)$.

Find:

(a) $|AB|$, $|BC|$, $|AC|$

(b) $\cos \langle \vec{AB}, \vec{AC} \rangle$, $\cos \langle \vec{BA}, \vec{BC} \rangle$, $\cos \langle \vec{CA}, \vec{CB} \rangle$

(c) the lengths of the medians AA_1 , BB_1 and CC_1 , where A_1 , B_1 and C_1 are the midpoints of BC , AC and AB respectively

(d) the area of the triangle ABC

17. Find the values of the parameter m for which the vectors $\vec{a} = m\vec{i} + 3\vec{j} + 4\vec{k}$ and $\vec{b} = 4\vec{i} + m\vec{j} - 7\vec{k}$ are orthogonal.

18. Find the volume of the tetrahedron $ABCD$ if

- (a) $A(1, 4, 1)$, $B(3, -7, -4)$, $C(0, 5, 0)$ and $D(1, 0, 3)$
- (b) $A(0, -3, 1)$, $B(2, -1, -1)$, $C(1, 6, 2)$ and $D(-1, 0, 0)$
19. Lie the following points in the same plane?
- (a) $A(3, 2, -1)$, $B(1, -6, -2)$, $C(10, -2, -3)$ and $D(2, -6, -4)$
- (b) $A(-3, 2, 4)$, $B(1, 2, -1)$, $C(0, 0, 3)$ and $D(2, 1, -1)$

Problem set 4.3

In Problems 1-6 find a vector parametric equation, scalar parametric equations and symmetric equations for the given lines.

- The line l through the point $M_0(2, 0, -7)$ and parallel to the vector $\vec{a} = \langle 3, -2, 4 \rangle$.
- The line l through the point $M_0(5, 8, 0)$ and parallel to the vector $\vec{a} = 2\vec{i} - 7\vec{k}$.
- The line l through the point $M_0(-3, 1, 5)$ and parallel to the line $(l_1): \frac{x+1}{-4} = \frac{y}{5} = \frac{z-5}{7}$.
- The line l through the points $M_1(1, 0, -5)$ and $M_2(-4, -7, 1)$.
- The line l through the points $O(0, 0, 0)$ and $M(1, 2, 3)$.
- The line l through the point $M_0(2, 0, -3)$ and parallel to the line (AB) where $A(1, 1, 1)$ and $B(0, -2, 5)$.

In Problems 7-8 find the points in which the given lines intersect the coordinate planes.

- The line l through the point $M_0(0, -1, 7)$ and parallel to the vector $\vec{a} = \langle 2, 1, 4 \rangle$.
- The line l through the points $M_1(2, -3, 4)$ and $M_2(5, 1, 3)$.

In Problems 9-12 determine whether the lines l_1 and l_2 are parallel, skew or intersecting.

9. $(l_1) : \frac{x-1}{1} = \frac{y}{2} = \frac{z+4}{3}$ and $(l_2) : \frac{x}{3} = \frac{y+5}{6} = \frac{z-1}{9}$
10. $(l_1) : \frac{x-1}{2} = \frac{y-4}{3} = \frac{z+1}{-5}$ and $(l_2) : \frac{x+3}{1} = \frac{y-2}{1} = \frac{z+19}{1}$
11. $(l_1) : \frac{x}{2} = \frac{y+1}{7} = \frac{z-3}{-5}$ and $(l_2) : \frac{x+1}{3} = \frac{y-2}{5} = \frac{z}{11}$
12. $(l_1) : \frac{x-2}{-3} = \frac{y+3}{7} = \frac{z}{9}$ and $(l_2) : \frac{x+5}{3} = \frac{z-1}{16}, y = -3$

In Problems 13-20 find the linear equation of the given planes.

13. The plane α through the point $M_0(10, 6, 3)$ with a normal the vector $\vec{n} = \langle 10, 10, -7 \rangle$.

14. The plane α through the point $M_0(-2, 9, 6)$ with a normal the vector $\vec{n} = \langle -6, 10, 4 \rangle$.

15. The plane α through the points $M_1(-8, -6, 7)$, $M_2(0, -12, 8)$ and $M_3(-1, -14, 1)$.

16. The plane α through the points $M_1(5, -9, 5)$, $M_2(9, -17, 4)$ and $M_3(-4, -10, 14)$.

17. The plane α through the point $M_0(2, 0, 3)$ which is parallel to the vectors $\vec{a} = \langle 1, 2, 5 \rangle$ and $\vec{b} = \langle -3, -1, 4 \rangle$.

18. The plane α through the point $M_0(-3, 1, 5)$ which is parallel to the vectors $\vec{a} = \langle 0, 1, 1 \rangle$ and $\vec{b} = \langle 1, -1, 3 \rangle$.

19. The plane α through the point $M_0(4, -2, 1)$ which is orthogonal to the planes $(\alpha_1) : x - 2y + z - 3 = 0$ and $(\alpha_2) : 3x - y + 2z - 4 = 0$.

20. The plane α through the points $M_1(3, -7, 2)$ and $M_2(1, 0, 4)$ which is parallel to the vector $\vec{a} = 5\vec{i} + 3\vec{j} - \vec{k}$.

In Problems 21-24 determine the angle between the given planes.

21. $(\alpha_1) : 5x + 4y - z + 1 = 0$ and $(\alpha_2) : 3x - y + 2z - 4 = 0$.

22. $(\alpha_1) : x + 9y - 2z - 11 = 0$ and $(\alpha_2) : 9x + y + 9z - 14 = 0$.

23. $(\alpha_1) : 2x - y + 3z - 7 = 0$ and $(\alpha_2) : 4x - 2y + 6z - 5 = 0$.

24. $(\alpha_1) : x - 3y - z + 1 = 0$ and $(\alpha_2) : 3x - 9y - 3z + 3 = 0$.

25. Find the distance from the point $M(5, 3, -2)$ to the plane $(\alpha) : 3x - 4y + 12z - 3 = 0$.

In Problems 26-32 determine the surfaces.

26. $y^2 + z^2 + 2y - z = 0$.

27. $x^2 + y^2 + z^2 - 6x + 2z + 9 = 0$.

28. $x^2 + 2y^2 - 4x + 16y = 0$.

29. $x^2 + 4y^2 + 16z^2 - 16 = 0$.

30. $x^2 + y^2 = 2(z + 1)^2$.

31. $\frac{x^2}{16} + \frac{y^2}{81} - \frac{(z - 1)^2}{4} = 1$.

32. $\frac{x^2}{1} + \frac{y^2}{4} = 4(z + 3)$.

4 Problems in Chapter 5

Problem set 5.1

In Problems 1-3 find the limit of the given vector function.

1. $\lim_{t \rightarrow 0} \langle \tan t, e^t, t^2 - 1 \rangle$

$$2. \lim_{t \rightarrow -1} \left\langle \frac{t^2 - t - 2}{t + 1}, \frac{t^2 + 4t + 3}{t + 1}, \frac{1 - t^2}{3t + 3} \right\rangle$$

$$3. \lim_{t \rightarrow 4} \left\langle \frac{\sqrt{t} - 2}{t^2 - 5t + 4}, \frac{t - 4}{\sqrt{t} - 2} \right\rangle$$

$$4. \text{ For } \vec{r}_1(t) = (t-2)\vec{i} + 2\sqrt{t}\vec{j} - t^3\vec{k} \text{ and } \vec{r}_2(t) = 2t^4\vec{i} - \frac{5}{t}\vec{j} + \frac{1+t}{1-t}\vec{k}$$

$$\text{find } \lim_{t \rightarrow 2} [\vec{r}_1(t) \cdot \vec{r}_2(t)] \text{ and } \lim_{t \rightarrow 2} [\vec{r}_1(t) \times \vec{r}_2(t)]$$

Problem set 5.2

In Problems 1-3 find the derivative $\vec{r}'(t)$ of the given vector function.

$$1. \vec{r}(t) = \frac{1}{t^3}\vec{i} + (t^7 + 12)\vec{j} + \frac{1}{2t-1}\vec{k}$$

$$2. \vec{r}(t) = \frac{t^2 + 1}{3t + 2}\vec{i} + t^2 \sin t \vec{j} + e^{t^2}\vec{k}$$

$$3. \vec{r}(t) = (2t + 5)^{100}\vec{i} + \sqrt{\frac{t+2}{t-1}}\vec{j}$$

In Problems 4-5 find the derivative $\vec{r}''(t)$ of the given vector function.

$$4. \vec{r}(t) = t \cos t \vec{i} + te^{t^2}\vec{j}$$

$$5. \vec{r}(t) = \frac{\cos 2t}{2}\vec{i} + (t^3 + \sin t)\vec{j} + t\sqrt{1+t^2}\vec{k}$$

In Problems 6-10 evaluate the integrals

$$6. \int \left(\sqrt[5]{t^3}\vec{i} + 5^t \cdot 8^t \vec{j} \right) dt$$

$$7. \int \left(\frac{\sin^3 t + 1}{\sin^2 t} \vec{i} + \frac{1}{3t+7} \vec{j} + \cos(2t-1)\vec{k} \right) dt$$

$$8. \int \left(t e^{3t} \vec{i} + t^3 \ln t \vec{j} + \tan^{-1} t \vec{k} \right) dt$$

$$9. \int_0^1 \left(t^3 \vec{i} + \sqrt[3]{t} \vec{j} + \frac{1}{1+t^2} \vec{k} \right) dt$$

$$10. \int_0^{\pi/2} \left(\cos^2 t \vec{i} + e^{2t} \cos t \vec{j} \right) dt$$

Problem set 5.3

In Problems 1-3 find the unit tangent vector $\vec{T}(t)$ to the curve given by the vector function $\vec{r}(t)$ at the indicated value of t .

$$1. \vec{r}(t) = 3t^2 \vec{i} + 2t \vec{j} + \frac{1}{t} \vec{k}, \quad t = 2$$

$$2. \vec{r}(t) = e^t \vec{i} + (5t + 1) \vec{j} + \sin t \vec{k}, \quad t = 0$$

$$3. \vec{r}(t) = \tan t \vec{i} + 2\sqrt{t} \vec{j} + 3 \cos t \vec{k}, \quad t = \pi$$

In Problems 4-6 find the scalar parametric equations for the tangent line to the curve given by the vector function $\vec{r}(t)$ at the indicated point.

$$4. \vec{r}(t) = t^4 \vec{i} + t^2 \vec{j} + t \vec{k}, \quad M(1, 1, 1)$$

$$5. \vec{r}(t) = t \ln t \vec{i} + t^3 \vec{j} + 4\sqrt{t} \vec{k}, \quad M(e, e^3, 4\sqrt{e})$$

$$6. \vec{r}(t) = t^2 \sin t \vec{i} + t \cos t \vec{k}, \quad M(0, 0, -\pi)$$

In Problems 7-8 find unit normal vectors $\vec{N}(t)$ and binormal vectors $\vec{B}(t)$ to the curve given by the vector function $\vec{r}(t)$ at the indicated values of t . Find the equations of the normal plane and the osculating plane.

$$7. \vec{r}(t) = 3 \sin t \vec{i} + 3 \cos t \vec{j} + 4t \vec{k}, \quad t_1 = 0, \quad t_2 = \pi, \quad t_3 = \pi/2$$

$$8. \vec{r}(t) = \cos^2 t \vec{i} + \sqrt{3} t \vec{j} + \sin t \cos t \vec{k}, \quad t_1 = \pi/4, \quad t_2 = 0$$

In Problems 9-14 find the length of the given curve.

$$9. \vec{r}(t) = 3 \cos t \vec{i} + 3 \sin t \vec{j}, \quad 0 \leq t \leq \pi$$

$$10. \vec{r}(t) = 2(t \sin t + \cos t) \vec{i} + 2(\sin t - t \cos t) \vec{j}, \quad 0 \leq t \leq 2\pi$$

$$11. \vec{r}(t) = a \cos^3 t \vec{i} + a \sin^3 t \vec{j}, \quad 0 \leq t \leq \pi/2, \quad a > 0$$

$$12. \vec{r}(t) = t^3 \vec{i} + \vec{j} + t^2 \vec{k}, \quad 0 \leq t \leq 2$$

$$13. \vec{r}(t) = (t \sin t + \cos t) \vec{i} + (\sin t - t \cos t) \vec{j} + 3t^2 \vec{k}, \quad 0 \leq t \leq 2\pi$$

$$14. \vec{r}(t) = 2t \vec{i} + \ln t \vec{j} - t^2 \vec{k}, \quad 1 \leq t \leq e$$

In Problems 15-17 find the curvature of the given curve at the indicated points.

$$15. \vec{r}(t) = a \cos t \vec{i} + b \sin t \vec{j}, \quad a > b > 0,$$

$$M_1 = (a, 0), \quad M_2 = (-a, 0), \quad M_3 = (0, b), \quad M_4 = (a\sqrt{2}/2, b\sqrt{2}/2)$$

$$16. \vec{r}(t) = 7 \sin t \vec{i} + 7 \cos t \vec{j} + t \vec{k}, \quad t \in \mathbb{R}$$

$$17. \vec{r}(t) = (t \sin t + \cos t) \vec{i} + (\sin t - t \cos t) \vec{j} + t^2 \vec{k},$$

$$M_1 = (-1, \pi, \pi^2), \quad M_2 = (\pi/2, 1, \pi^2/4)$$

In Problems 18-21 find the velocity, acceleration, and speed of an object with the given position vector function at the indicated value of t .

$$18. \vec{r}(t) = t \vec{i} + (5 - t^2) \vec{j}, \quad t = 3$$

$$19. \vec{r}(t) = 2\sqrt{t} \vec{i} + t \ln t \vec{j}, \quad t = 1$$

$$20. \vec{r}(t) = \cos^3 t \vec{i} + \sin^3 t \vec{j} + \cos 2t \vec{k}, \quad t = \pi/4$$

$$21. \vec{r}(t) = \cos^2 t \vec{i} + \sin t \cos t \vec{j} + \sin t \vec{k}, \quad t = \pi/2$$

5 Problems in Chapter 6

Problem sets 6.1 and 6.2

In Problems 1-6 find the domain of the given function.

$$1. z = 5x - 4y - 7$$

$$2. z = \sqrt{3x^2 + 3y^2}$$

$$3. z = \frac{2x + y}{x^2 + y^2 - 9}$$

$$4. z = \frac{\sqrt{x - y^2}}{1 - y^2}$$

$$5. u = z + \ln(x - y)$$

$$6. u = \sqrt{2 - x^2 + y^2 + z^2}$$

In Problems 7-10 sketch the graph of the given function.

$$7. f(x, y) = 1$$

$$8. f(x, y) = x$$

$$9. f(x, y) = x^2$$

$$10. f(x, y) = x^2 + \frac{1}{2}y^2$$

In Problems 11-18 find the limit, if it exists, or show that the limit does not exist.

$$11. \lim_{(x, y) \rightarrow (-1, 1)} (x^2 y^4 - 3x^4 + 7x^3 y)$$

$$12. \lim_{(x, y) \rightarrow (\pi/2, \pi/4)} y^2 \sin(y - x)$$

$$13. \lim_{(x, y) \rightarrow (0, 0)} \frac{y^2 - x^2}{x^2 + y^2}$$

$$14. \lim_{(x, y) \rightarrow (0, 0)} \frac{\sqrt{xy}}{x + y}$$

$$15. \lim_{(x, y) \rightarrow (0, 0)} \frac{5yx^2}{x^2 + y^2}$$

$$16. \lim_{(x, y) \rightarrow (0, 0)} \frac{y^5 - x^5}{x^2 + y^2}$$

$$17. \lim_{(x, y, z) \rightarrow (1, 0, 1)} (y + z)e^{xz}$$

$$18. \lim_{(x, y, z) \rightarrow (0, 0, 0)} \frac{xz + xy^2 + yx^2}{x^3 + y^3 + z^3}$$

In Problems 19-22 determine the points at which the function is continuous.

$$19. f(x, y) = 7x^2 - 4y^2 + 3xy - 4y^3$$

$$20. f(x, y) = \frac{x + y}{x - y}$$

$$21. f(x, y, z) = \frac{\sqrt{x}}{x^2 + y^2 - z^2}$$

$$22. f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$$

Problem set 6.3

In Problems 1-6 find the derivatives f_x and f_y of the given functions.

$$1. f(x, y) = x^3 - y^2 + 5xy^4 - 3x^2y$$

$$2. f(x, y) = x + y + \frac{x}{y}$$

$$3. f(x, y) = 4x\sqrt{y} + \sin(x + 3y)$$

$$4. f(x, y) = \frac{x^2 + 2y}{5x - 3y}$$

$$5. f(x, y) = e^{\frac{x+y}{x-y}}$$

$$6. f(x, y) = \sqrt[3]{xy - y^2}$$

In Problems 7-10 find the derivatives f_x , f_y and f_z of the given functions.

$$7. f(x, y, z) = xyz - \frac{x^2 + y^2}{z^2}$$

$$8. f(x, y, z) = \ln(x^3 + y^3 - xz)$$

$$9. f(x, y, z) = \tan(5x^2y - zy^2 + xz^2)$$

$$10. f(x, y, z) = 3^{xyz} + \frac{xy}{z}$$

In Problems 11-16 find all derivatives of the second order for the given functions.

11. $f(x, y) = x^2y + y^2x + \frac{y}{x}$

12. $f(x, y) = e^{2x+y}$

13. $f(x, y) = \ln\left(\frac{x}{y}\right)$

14. $f(x, y) = x \tan^{-1}\left(\frac{y}{x}\right)$

15. $f(x, y, z) = x^2y + y^2z + z^2x - 3xyz$

16. $f(x, y, z) = \frac{x + y + z}{xyz}$

In Problems 17-19 find the indicated derivative.

17. $f(x, y) = x^2 - xy^4 + 3x^3y^2, \quad f_{xyx}$

18. $f(x, y) = \sin(3x + 2y^2), \quad f_{yyxy}$

19. $f(x, y, z) = \ln(x^2 + y^2 + z^2), \quad f_{xyz}$

Problem set 6.4

In Problems 1-6 find the total differential.

1. $f(x, y) = x^3 - xy^2 + x^2y - y^3$

2. $f(r, s) = e^{rs} + e^{r-s}$

3. $f(u, v) = \ln\sqrt{u^2 + v^2}$

4. $f(x, y) = \sin^2(x^3 + y^3)$

5. $f(x, y, z) = x^2 + y^2 - z^2 - xy + yz$

6. $f(x, y, z) = z \tan^{-1}\left(\frac{y}{x}\right)$

In Problems 7-8 compute approximately using differentials.

7. $\sqrt{(2.02)^2 + 4(1.97)^2 - (1.98)^2}$

8. $\sqrt[4]{(5.1)^2 - (2.99)^2}$

9. Find, approximately, the volume of a right circular cylinder of height 10.1 cm and radius of base 8.9 cm.

In Problems 10-13 use the Chain Rule to find dz/dt .

$$10. z = x^2 - 3y^2 + xy, \quad x = e^t, \quad y = t^2 - 3t + 5$$

$$11. z = \sqrt{x^2 - 4y^2}, \quad x = \sin t, \quad y = \cos t$$

$$12. z = \tan\left(\frac{x}{y}\right), \quad x = \sqrt{t}, \quad y = \sqrt[3]{t} + 2$$

$$13. z = \ln^2(2x + 3y), \quad x = t^5, \quad y = \tan t$$

In Problems 14-17 use the Chain Rule to find the partial derivatives.

$$14. z = e^{2x-3y}, \quad x = \tan(t + s^2), \quad y = t^2 - s$$

$$15. z = u^2 \ln v, \quad u = \frac{y}{x}, \quad v = x^2 + y^2$$

$$16. z = u^2 v - v^2 u, \quad u = x \sin y, \quad v = y \cos x$$

$$17. u = x^2 + y^2 + z^2, \quad x = te^s, \quad y = t^2 + s^2, \quad z = \frac{t}{s}$$

In Problems 18-19 find $y'(x)$ for the function $y = y(x)$ defined by the equation

$$18. x^3 + x^2 y^4 = 1 + y^2 e^{x^3}$$

$$19. \sin(x + 2y) = x \cos y$$

In Problems 20-21 find $\partial z/\partial x$ and $\partial z/\partial y$ for the function $z = z(x, y)$ defined by the equation

20. $x^3 + y^3 - z^2 = 2xyz$

21. $xy\sqrt{z} = \sin(x + y + z)$

Problem set 6.5

In Problems 1-4 find the gradient vector.

1. $f(x, y) = x^2y^2 - y^3$

2. $f(x, y) = \sin(x + 3y) + y^2$

3. $f(x, y, z) = \ln(x^2 + y^2 + z^2)$

4. $f(x, y, z) = x^2 \tan(y^4 z)$

In Problems 5-8 find the directional derivatives of the function at the given point in the direction of the indicated vector.

5. $f(x, y) = 3 + 2y\sqrt{x}$, $(9, 1)$, $\vec{u} = \langle 3, -2 \rangle$

6. $f(x, y) = \sin(x^2 + y^2)$, $(\sqrt{\pi}, \sqrt{\pi})$, $\vec{u} = \langle -2, 7 \rangle$

7. $f(x, y, z) = \frac{y}{2x + 2z}$, $(3, 1, 0)$, $\vec{u} = \langle 2, 3, 4 \rangle$

8. $f(x, y, z) = \frac{x}{y} + \frac{y}{z}$, $(0, 1, 2)$, $\vec{u} = \langle -5, 5, 2 \rangle$

In Problems 9-12 find equations of the tangent plane and the normal line to the given surface at the indicated point.

9. $z = x^2 + y^2, \quad (2, 1, 5)$

10. $x^2 + y^2 + z^2 - 2x + 4y + 6z + 5 = 0, \quad (3, 0, -2)$

11. $xyz = x^2 + 2y - 2, \quad (1, 1, 1)$

12. $x^2 - 2y^2 + z^2 + xz = 2, \quad (2, 1, -2)$

In Problems 13-18 find the local maximum and minimum values of the given functions.

13. $f(x, y) = x^2 - xy + y^2$

14. $f(x, y) = xy + x - y - 1$

15. $f(x, y) = x^2 + xy + y^2 - 3x - 6y$

16. $f(x, y) = 2x - 4y - x^2 - 2y^2$

17. $f(x, y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$

18. $f(x, y) = x^3 + 2y^3 - 216y - 18x$

19. Find, the point on the plane $x - y + 2z - 6 = 0$ at minimum distance from the origin.

20. A rectangular box is to be inscribed in the cone $z = 3 - \sqrt{x^2 + y^2}$, $z \geq 0$. Find, the dimensions for the box that maximize its volume.

6 Problems in Chapter 7

Problem set 7.1

In Problems 1-12 evaluate the double integral.

1. $\int \int_D xy^2 dx dy, \quad D = \{(x, y) \mid 0 \leq x \leq 3, 1 \leq y \leq 2\}$

2. $\int \int_D \frac{3x^2}{1+y^2} dx dy$, $D = \{(x, y) \mid 0 \leq x \leq 1, -1 \leq y \leq 1\}$
3. $\int \int_D e^{y/x} dx dy$, $D = \{(x, y) \mid 0 \leq x \leq 1, 0 \leq y \leq x^2\}$
4. $\int \int_D \sqrt{xy - y^2} dx dy$, $D = \{(x, y) \mid y \leq x \leq 2y, 0 \leq y \leq 2\}$
5. $\int \int_D x^2 y dx dy$, D is enclosed by $y = 0$ and $y = \sqrt{1 - x^2}$
6. $\int_0^1 \int_y^1 y \cos(2x^3) dx dy$
7. $\int \int_D (x + y) dx dy$, D is enclosed by $x = 0$, $y = 0$ and $x + y = 5$
8. $\int \int_D (x - y) dx dy$, D is enclosed by $y = 0$, $y = x$ and $x + y = 2$
9. $\int \int_D e^{x^2+y^2} dx dy$, $D = \{(x, y) \mid x^2 + y^2 \leq 1\}$
10. $\int \int_D (x^2 + y^2)^{5/2} dx dy$, $D = \{(x, y) \mid x^2 + y^2 \leq a^2, y \geq 0\}$, $(a > 0)$
11. $\int \int_D \frac{\ln(x^2 + y^2)}{x^2 + y^2} dx dy$, $D = \{(x, y) \mid 1 \leq x^2 + y^2 \leq e^2\}$
12. $\int \int_D \sin \sqrt{x^2 + y^2} dx dy$, $D = \{(x, y) \mid \pi^2 \leq x^2 + y^2 \leq 4\pi^2\}$

Problem set 7.2

1. Find the volume under the plane $x + y + z = 6$, above the plane $z = 0$, and with sides $y = 0$, $y = x$ and $x = 3$.

2. Find the volume of the solid under the plane $2 - x - y - 2z = 0$, above the plane $z = 0$, and with sides $y = x$ and $y = x^2$.
3. Find the volume of the solid under the paraboloid $z = 4 - x^2 - y^2$, and between the planes $x = \pm 1$ and $y = \pm 1$.
4. Find the volume of the solid bounded by surfaces $z = 5$, $(x^2 + y^2)^2 = 2xy$, $z = 0$ that lies in the first octant.
5. Find the area of the part of the plane $x/2 + y/3 + z/4 = 1$, that lies in the first octant.
6. Find the area of the part of the surface $2z = x^2 - y^2$, that is inside the cylinder $x^2 + y^2 = 1$.
7. Find the area of the part of the cylinder $x^2 + z^2 = 4$, that is inside the cylinder $x^2 + y^2 = 4$.
8. Find the area of the part of the cone $z = \sqrt{x^2 + y^2}$, that is inside the cylinder $x^2 + y^2 - 2ax = 0$.
9. Find the area of the part of the sphere $z = \sqrt{a^2 - x^2 - y^2}$, that is inside the cylinder $x^2 + y^2 = b^2$, $0 < b < a$.
10. Find the area of the part of the cylinder $z^2 + x^2 = a^2$, that is above the triangle bounded by $x = 0$, $y = 0$, $x + y = a$.

Problem set 7.3

In Problems 1-10 evaluate the triple integral.

1.
$$\int \int \int_R xyz \, dx \, dy \, dz, \quad R = \{(x, y, z) \mid 0 \leq x \leq 2, 1 \leq y \leq 5, 0 \leq z \leq 3\}$$

2. $\int \int \int_R x^3 y^2 z \, dx \, dy \, dz$, $R = \{(x, y, z) \mid 0 \leq z \leq xy, 0 \leq y \leq x, 0 \leq x \leq 1\}$

3. $\int \int \int_R dx \, dy \, dz$, R is bounded by $x + y + z = a$ and the coordinate planes

4. $\int \int \int_R (x^2 + y^2) \, dx \, dy$, $R = \{(x, y, z) \mid 0 \leq z \leq y + x^2, x \leq y \leq \sqrt{x}, 0 \leq x \leq 1\}$

5. $\int \int \int_R (1 + z) \, dx \, dy \, dz$, R is in the first octant inside the cylinders $x = 4 - z^2$ and $y = 4 - z^2$

6. $\int \int \int_R e^{x-y-z} \, dx \, dy \, dz$, $R = \{(x, y, z) \mid 0 \leq y \leq x - z, 0 \leq z \leq x, 0 \leq x \leq 1\}$

7. $\int \int \int_R z \sqrt{x^2 + y^2} \, dx \, dy \, dz$, $R = \{(x, y, z) \mid 0 \leq y \leq \sqrt{2x - x^2}, 0 \leq z \leq 1\}$

8. $\int \int \int_R y^2 \, dx \, dy \, dz$, R is bounded by the cone $z = \sqrt{x^2 + y^2}$ and the plane $z = 1$

9. $\int \int \int_R \sqrt{x^2 + y^2 + z^2} \, dx \, dy \, dz$, $R = \{(x, y, z) \mid x^2 + y^2 + z^2 \leq x\}$

10. $\int \int \int_R \sin(x^2 + y^2 + z^2)^{3/2} \, dx \, dy \, dz$, $R = \{(x, y, z) \mid x^2 + y^2 + z^2 \leq 4\}$

11. Find the volume of the solid bounded by the planes $x + y + z = 4$, $y = 3$, $x = 2$, $x = 0$, $y = 0$ and $z = 0$.

12. Find the volume of the solid that lies inside the sphere $x^2 + y^2 + z^2 = b^2$ and outside the cylinder $x^2 + y^2 = a^2$, ($0 < a < b$.)
13. Find the volume of the solid under the cone $z = \sqrt{2x^2 + 2y^2}$, bounded below by the plane $z = 0$, and within the cylinder $x^2 + y^2 = 4x$.
14. Find the volume of the solid that lies between the spheres $x^2 + y^2 + z^2 = a^2$ and $x^2 + y^2 + z^2 = b^2$, ($0 < a < b$.)